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Kinematical properties of the Compton effect

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Abstract. It is shown that, in the Compton effect, the velocity of the recoil electron is obtained by adding relativistically the projections of the photon velocities.

1. Introduction

It is well known that, in the Compton effect, the shift in wavelength of the scattered light has been explained by considering a particle-like collision between a photon and an electron. The Compton formula, which allows us to calculate the shift in wavelength, refers both to the Planck constant and to the electron mass. The presence of these fundamental constants validates in some manner the particle model involving photons.

Ashworth and Jennison (1974) have shown that the Compton effect could have a purely kinematical interpretation in relativistic mechanics. They established a relation between the frequencies of the incident and the scattered x rays without the presence of the Planck constant h or the electron mass m. Working in the same direction, we show that even the frequencies of the incident and of the scattered x rays are not necessarily present.

The Compton effect can be characterised as a purely kinematical phenomenon involving the relativistic addition of velocities of the incident and scattered photons.

2. Known kinematical properties

Starting from the Compton equations (Compton 1923)

$$h\nu + m_0c^2 = h\nu' + mc^2 \tag{1}$$

$$h\boldsymbol{\nu}/c = h\boldsymbol{\nu}/c + m\boldsymbol{v} \tag{2}$$

we call ϕ the deviation angle of the scattered light with frequency ν' and ψ the deviation angle of the recoil electron (figure 1).

When projecting equation (2) on directions parallel and perpendicular to the incident photon we obtain

$$h(\nu/c) = h(\nu'/c)\cos\phi + mv\cos\psi \tag{3}$$

$$0 = h(\nu'/c)\sin\phi - mv\sin\psi. \tag{4}$$

The functional Compton equation is

$$\cos\phi = 1 - \frac{m_0c^2}{h}\left(\frac{1}{\nu'} - \frac{1}{\nu}\right). \tag{5}$$

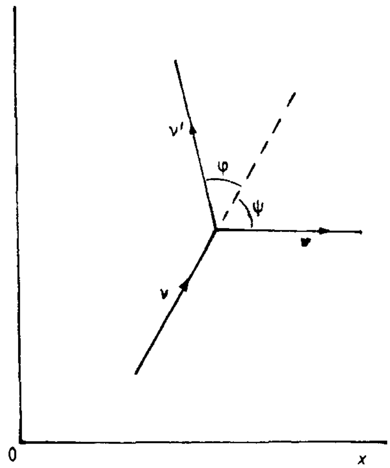

Figure 1. Collision between a photon and an electron.

By combining (1) and (3) we obtain

$$h(\nu'/c)(1-\cos\phi)=(m_0-m)c+mv\cos\psi \tag{6}$$

which gives with (5)

$$\frac{\nu'}{\nu}=\frac{1-(v/c)\cos\psi}{[1-(v/c)^2]^{1/2}}. \tag{7}$$

This equation was established by Ashworth and Jennison in 1974. They also showed that Compton scattering can be explained as a pure specular reflection in the proper frame of reference of the scattered element (Ashworth and Jennison 1974).

3. New kinematical properties

3.1. Symmetrical role of ν and ν'

When starting from these results, we remark that, according to light propagation laws, the incident and the scattered x rays must play an entirely symmetrical role. We may then expect to find a relation equivalent to (7) where the role of ν and ν' are inverted, and the angle ψ between the incident photon and the electron replaced by the angle $\phi+\psi$ between the scattered photon and the electron.

In order to establish this relation, we can project equation (2) on directions parallel and perpendicular to the recoil electron. We obtain

$$h(\nu/c)\cos\psi=h(\nu'/c)\cos(\phi+\psi)+mv \tag{8}$$

$$h(\nu/c)\sin\psi=h(\nu'/c)\sin(\phi+\psi). \tag{9}$$

This gives

$$h^2\nu^2/c^2=h^2\nu'^2/c^2+2h\nu'm(v/c)\cos(\phi+\psi)+m^2v^2. \tag{10}$$

By combining it with

$$m^2v^2=m^2c-m_0^2c^2$$

we find

$$\frac{\nu}{\nu'} = \frac{1-(v/c)\cos(\phi+\psi)}{[1-(v/c)^2]^{1/2}}. \tag{11}$$

This equation is symmetrical with equation (7). We can obtain it directly from the system (1) and (2) by expressing the proper energy of the electron + photon couple as unchanged by the collision:

$$(h\nu + m_0c^2)^2 - h^2\nu^2 = (h\nu' + mc^2)^2 - (h\boldsymbol{\nu}'/c + m\boldsymbol{v})^2c^2 \tag{12}$$

$$\nu m_0 = m(\nu' - \boldsymbol{\nu}' \cdot \boldsymbol{v}/c). \tag{13}$$

As

$$\boldsymbol{\nu}' \cdot \boldsymbol{v} = \nu' v \cos(\phi+\psi)$$

we obtain (11).

3.2. Relativistic addition of velocities

In addition to embracing the kinematical property of the Compton effect as a usual light specular reflection in the proper frame of the electron, equation (11) leads to a new kinematical property.

When we multiply (7) and (11) we obtain

$$1 - \frac{v^2}{c^2} = \left(1 - \frac{v}{c}\cos\psi\right)\left(1 - \frac{v}{c}\cos(\phi+\psi)\right)$$

which leads to

$$v = c\frac{\cos\psi + \cos(\phi+\psi)}{1+\cos\psi\cos(\phi+\psi)}. \tag{14}$$

If we call

$$v_1 = c\cos\psi \tag{15}$$

the projection of the incident photon velocity along the direction of the recoil electron and

$$v_2 = c\cos(\phi+\psi) \tag{16}$$

the projection of the scattered photon velocity along the same direction, equation (14) becomes

$$v = \frac{v_1 + v_2}{1 + v_1v_2/c_2}. \tag{17}$$

In the Compton effect, the velocity of the electron after collision is obtained by adding relativistically the projection along the direction of the recoil electron of the incident and scattered photon velocities.

This kinematical property is remarkable as it is independent not only of the Planck constant and the electron mass, but also from the photon frequencies ν and ν'.

4. Conclusion

Even though the Compton effect shows that light behaves like particles when interacting with matter in photon-electron collisions, it also shows that some wave properties are still present and cannot be ignored.

This result may be considered together with recent symmetrical investigations of wave properties associated with the Compton wavelength of material particles (Jennison 1983, Elbaz 1983, 1984, 1985, 1986).

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